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| *Title:* | **AhG7: Overflow Prevention in HEVC inverse transform** | | |
| *Status:* | Input Document to JCT-VC | | |
| *Purpose:* | Proposal | | |
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# Abstract

Modification for HEVC inverse transform framework which allows preventing overflow of 16 bits in 3 temporal buffers by using no more than 2 clipping operations. Proposed modification guarantees all temporal buffers within 16 bits even for random input for inverse transform (non-confirmed bit streams). Performance test show no change under common test conditions due to proposed modification.

Additionally 32 bit overflow in intermediate calculation for JCTVC-F251 core transform is studied here. The same solution with no more than 2 clipping operations per pixel guarantees no 32 bits overflow in register for JCTVC-F251. Performance of JCTVC-F251 doesn’t change due to proposed modification.

Additional test results for 12 bits internal bit-depth show noise level change of BD-rate (0.00% in average) due to proposed change for both HM4.0 and JCTVC-F251 inverse transforms.

# Introduction

HM4.0 inverse transform [1] consist of 3 steps: de-quantizer, 1st (column) and 2nd (row) inverse transform. After each step data are storied in temporal buffer of TU size. This is desirable for the standard to have all temporal buffers within 16 bits. For non-confirmed bit-streams overflow of 16 bits may happen for all 3 buffers (refer to examples below). So in the worst case 3 clipping operations per pixel are required in inverse transform framework to keep all temporal buffers within 16 bits.

# HM4.0 inverse transform framework

The dequantizer is specified as follows

coeffQ = ((level\*IQ << (QP/6)) + offset)>>(M-1+(B-8)), offset = 1<<(M-2+(B-8)),

coeffQ = min(32767,max(-32768,coeffQ)). (1)

Here level is qunatized coefficents, M is

B = internal bit depth (as specified by InternalBitDepth in the common config files)

W,H = transform horizontal and vertical sizes

M = log2(W\*H)/2

The clipping ensures that the value of the quantized transform coefficients, *coeffQ*, can be represented with 16 bits. The scaling is chosen so that clipping only occur for extreme input values and/or unreasonable quantizer behaviour.

The inverse transform is specified as a matrix multiplication in two stages, horizontal and vertical.

The first stage is column transform:

.

After the first stage, the resulting transform coefficients are de-scaled as follows

temp = (register1 + offset1)>>shift1 (2)

shift1 = 7; offset1 = 1<<( shift1-1).

The first stage is row transform:

.

After the second stage, the resulting transform coefficients are scaled as follows

rec\_residual = ( + offset2)>>shift2; (3)

shift2=(12-(B-8)),offset2 = 1<<( shift2-1).

Transform for sizes N=4,…,32 matrices (both approximating DCT and DST) can be found in appendix A.

As described here in current HM4.0 implementation we have one clipping after de-quantizer. In matrix multiplication form of HM4.0 inverse transform intermediate buffer between 1st and 2nd transform is 32 bits. In “partial butterfly branch” intermediate buffer is 16 bits and at final stage both 1st and 2nd inverse transforms use conversion of 32 bits intermediate values to 16 bits output in partial butterfly branch. This is compiler dependent cast. This 32 to 16 bits conversion works as clipping (at least in the sense that output value is quarantined to be within 16 bits).

One of the goals of this contribution is to define dynamic range for temporal buffers for HEVC specification (we propose to use 16 bits temporal buffers) and clearly specify how to process transform output if it exceed 16 bits range.

# Dynamic range study method

As it is specified in section 2 both horizontal and vertical transforms are matrix multiplication. For example, first (column) transform is

(4)

If the maximum absolute value of one multiplier is limited then

. (5)

Here is maximum among all columns of absolute values sums in transform matrices. L1-norms for all inverse transforms used in HM4.0 are shown in Table 1.

**Table 1.** L1 norms for1D inverse transforms in HM4.0.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Transform | invDST4 | invDCT4 | invDCT8 | invDCT16 | invDCT32 |
|  | 242 | 247 | 479 | 940 | 1862 |

L1-norm is the maximum factor for magnitude increment during 1D transform. More over if are arbitrary numbers in diapason then it is always possible to find example then will reach upper limit .

Upper limit for magnitude in temporal buffer is

. (6)

The same analysis is valid for the second (row) transform. If then

(7)

and

, (8)

For convenience excel spreadsheet is attached to this contribution. Page “NoClipInverseTransfomr” shows dynamic range of data on each stage of inverse transform in the worst case.

# Overflow examples

Let’s analyze 4x4 inverse transform for simplicity. Let all 16 elements in coeffQ matrix are equal to then

.

All elements in first row exceed 16 bits diapason.

If internal dynamic range is 12 bits (B=12) then

Upper-left cell of this matrix contains element which is out of 16 bits diapason.

This example shows that without dynamic range restriction after 1st and 2nd inverse transform we may have 16 bits overflow in both temporal buffers.

If there is a clip to 16 bits diapason after 1st transform then in this example:

.

If internal bit-depth is 13 bits then we meet overflow after 2nd inverse transform again:

.

So in this example high internal bit depth was chosen to demonstrate overflow for smallest transform size 4x4. In larger TUs overflow of 2nd transform output may happen even for internal bit-depth 8.

We have to mention that example of coeffQ we use here cannot be output of forward HM4.0 transform. But it may come from non-confirmed bit-stream. Our goal is to prevent overflow even for the worst case minimizing clip operations.

# JCTVC-F251 core transform

HM4.0 transform can be implemented in two equivalent forms: matrix multiplication and partial butterfly. In [2] the modification of [1] was proposed; it allows additionally represent transform in full-factorized form which significantly reduces the number of multiplications compare to partial butterfly form. The difference between [1] and [2] is only shift1=(7+6) and shift2 = values in (2)-(3) and . Transform matrices for [2] can be found in appendix B. Corresponding L1-norms are summarized in Table 2.

**Table 2**. L1-norms for1D inverse transforms in JCTVC-F251.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Transform | invDST4 | invDCT4 | invDCT8 | invDCT16 | invDCT32 |
|  | 15488 | 15808 | 30622 | 30622 | 119262 |

Dynamic range analysis at any stage of inverse transform for [2] is also included into attached excel spread-sheet. As it was mentioned above this is absolutely the worst case analysis. In this absolutely the worst case (not for output of correct forward transform) an overflow may happen not only for temporal buffers but an overflow of 32 bit register is also possible.

This proposal targets to resolve this issue and guarantee no overflow both 16 bits temporal buffers and 32 bits registers (intermediate calculations) for the absolutely the worst case (cannot happen if correct forward transform is used, may comes from non-confirmed bit-streams only).

# Proposed solution (no more than 2 clipping)

Let’s assume that output of de-quantize is limited . This can be achieved by clipping after de-quantizer as in HM4.0, by encoder constrain as it was proposed in [4] or by level limit described in [4 = Sharp+Samsung contribution in upcoming meeting]. Suppose we insert only one additional clipping after 1st .

Let’s find and which guarantee no overflow at any stage of inverse transform.

Form requirements

,

and (7)-(8) come to

, (9).

By choosing accordingto (9) we guarantee both: register during second inverse transform is inside 32 bits diapason and output of second inverse transform is within 16 bits.

Form requirements

,

and (5)-(6) we come to

, (10).

By choosing accordingto (10) we guarantee both: register during first inverse transform is inside 32 bits diapason and temporal buffer between 1st and 2nd transforms is within 16 bits.

Because shift1 doesn’t depend on internal bit-depth in HM4.0 framework the limit also depends on transform size only. The limit depends both on internal bit-depth and transform size.

Limit values and for HM4.0 core transform are summarized in Tables 3-4.

**Table 3.** Upper limit for magnitude of de-quantized coefficients in HM4.0 core transform kernel (Size here is the size of first 1D inverse transform)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Size | 4(DST) | 4(DCT) | 8 | 16 | 32 |
| Max1 | 32767 | 32767 | 32767 | 32767 | 32767 |

**Table 4.** Upper limit for magnitude of temporal buffer values after 1st inverse transform in HM4.0 core transform kernel (Size here is the size of 2nd 1D inverse transform).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Internal bit-depth | 4(DST) | 4(DCT) | 8 | 16 | 32 |
| 14 | 8665 | 8490 | 4378 | 2230 | 1126 |
| 13 | 17331 | 16980 | 8756 | 4461 | 2252 |
| 12 | 32767 | 32767 | 17512 | 8923 | 4505 |
| 11 | 32767 | 32767 | 32767 | 17847 | 9010 |
| 10 | 32767 | 32767 | 32767 | 32767 | 18020 |
| 9 | 32767 | 32767 | 32767 | 32767 | 32767 |
| 8 | 32767 | 32767 | 32767 | 32767 | 32767 |

Limit values and for JCTVC-F251 core transform are summarized in Table 5-6.

**Table 5.** Upper limit for magnitude of de-quantized coefficients in F-251 core transform kernel (Size here is the size of first 1D inverse transform)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Size | 4(DST) | 4(DCT) | 8 | 16 | 32 |
| Max1 | 32767 | 32767 | 32767 | 32767 | 18006 |

**Table 6.** Upper limit for magnitude of temporal buffer values after 1st inverse transform F-251 core transform kernel (Size here is the size of 2nd 1D inverse transform).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Internal bit-depth | 4(DST) | 4(DCT) | 8 | 16 | 32 |
| 14 | 8665 | 8490 | 4382 | 2224 | 1125 |
| 13 | 17331 | 16980 | 8765 | 4449 | 2250 |
| 12 | 32767 | 32767 | 17531 | 8899 | 4501 |
| 11 | 32767 | 32767 | 32767 | 17798 | 9002 |
| 10 | 32767 | 32767 | 32767 | 32767 | 18005 |
| 9 | 32767 | 32767 | 32767 | 32767 | 18006 |
| 8 | 32767 | 32767 | 32767 | 32767 | 18006 |

Excel spreadsheet attached to this contribution has one more page: “NoMoreThan2CllipsBalancedTransf” which demonstrates dynamic range on each stage of inverse transform process with proposed modification of the framework.

# Proposed WD changes

There is trasnform part description in WD so far but HM4.0 core trasnform framework is described section 2.

Proposed here changes will modify this description as follows (chanegs hig-lighted in yellow).

================description start==========================

Inverse transform consists of 3 steps: dequantizer, 1st (column) and second (row) inverse transforms. Output of each step is storied in TU size temporal buffer of 16 bits size. Intermediate calculation requires 32 bits precision.

The dequantizer is specified as follows

coeffQ = ((level\*IQ << (QP/6)) + offset)>>(M-1+(B-8)), offset = 1<<(M-2+(B-8)),

coeffQ = min(Max1,max(-Max1,coeffQ)).

Here level is qunatized coefficents, M is

B = internal bit depth (as specified by InternalBitDepth in the common config files)

W,H = transform horizontal and vertical sizes

M = log2(W\*H)/2

The clipping ensures that the value of the quantized transform coefficients, *coeffQ*, can be represented with 16 bits. The scaling is chosen so that clipping only occur for extreme input values and/or unreasonable quantizer behaviour.

The inverse transform is specified as a matrix multiplication in two stages, horizontal and vertical.

The first stage is column transform:

.

After the first stage, the resulting transform coefficients are de-scaled as follows

temp = min(Max2,max(-Max2, (register1 + offset1)>>shift1))

shift1 = 7; offset1 = 1<<( shift1-1).

The first stage is row transform:

.

After the second stage, the resulting transform coefficients are scaled as follows

rec\_residual = ( + offset2)>>shift2;

shift2=(12-(B-8)),offset2 = 1<<( shift2-1).

Here Max1 and Max2 are specified by (9) and (10) correspondently.

================end of description==========================

Depending on which core transform will be chosen for HM5.0 [1] or [2] mentioned here Max1 and Max2 are specified by Tables 3-4 or tables 5-6 correspondently.

# Results

Proposed changes were tested both for HM4.0 and F-251 core transforms. No performance changes for both were found under common test condition described in JCTVC-F900 [4].

Additional test was dome for internal bit-depth = 12. As it shown in Tables 4-6 limitation becomes tighter if internal bit-depth goes higher. In this case small performance variation was found, but in average BD-rate change is 0.0%.

Summarized results are given in Table 7-10.

Table 7. Modified clipping in HM4.0 (common test conditions)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **All Intra HE** | | | **All Intra LC** | | |
|  | Y | U | V | Y | U | V |
| Class A | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class B | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class C | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class D | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class E | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class F | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| **Overall** | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
|  | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Enc Time[%] | 101% | | | 103% | | |
| Dec Time[%] | 104% | | | 104% | | |
|  |  |  |  |  |  |  |
|  | **Random Access HE** | | | **Random Access LC** | | |
|  | Y | U | V | Y | U | V |
| Class A | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class B | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class C | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class D | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class E |  |  |  |  |  |  |
| Class F | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| **Overall** | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
|  | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Enc Time[%] | 100% | | | 101% | | |
| Dec Time[%] | 100% | | | 102% | | |
|  |  |  |  |  |  |  |
|  | **Low delay B HE** | | | **Low delay B LC** | | |
|  | Y | U | V | Y | U | V |
| Class A |  |  |  |  |  |  |
| Class B | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class C | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class D | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class E | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class F | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| **Overall** | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
|  | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Enc Time[%] | 100% | | | 101% | | |
| Dec Time[%] | 102% | | | 103% | | |
|  |  |  |  |  |  |  |
|  | **Low delay P HE** | | | **Low delay P LC** | | |
|  | Y | U | V | Y | U | V |
| Class A |  |  |  |  |  |  |
| Class B | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class C | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class D | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class E | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class F | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| **Overall** | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
|  | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Enc Time[%] | 100% | | | 104% | | |
| Dec Time[%] | 101% | | | 101% | | |

Table 8. Modified clipping with F251 (common test conditions)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **All Intra HE** | | | **All Intra LC** | | |
|  | Y | U | V | Y | U | V |
| Class A | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class B | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class C | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class D | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class E | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class F | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| **Overall** | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
|  | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Enc Time[%] | 99% | | | 100% | | |
| Dec Time[%] | 98% | | | 97% | | |
|  |  |  |  |  |  |  |
|  | **Random Access HE** | | | **Random Access LC** | | |
|  | Y | U | V | Y | U | V |
| Class A | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class B | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class C | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class D | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class E |  |  |  |  |  |  |
| Class F | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| **Overall** | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
|  | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Enc Time[%] | 99% | | | 99% | | |
| Dec Time[%] | 98% | | | 97% | | |
|  |  |  |  |  |  |  |
|  | **Low delay B HE** | | | **Low delay B LC** | | |
|  | Y | U | V | Y | U | V |
| Class A |  |  |  |  |  |  |
| Class B | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class C | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class D | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class E | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class F | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| **Overall** | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
|  | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Enc Time[%] | 99% | | | 99% | | |
| Dec Time[%] | 97% | | | 95% | | |
|  |  |  |  |  |  |  |
|  | **Low delay P HE** | | | **Low delay P LC** | | |
|  | Y | U | V | Y | U | V |
| Class A |  |  |  |  |  |  |
| Class B | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class C | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class D | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class E | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Class F | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| **Overall** | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
|  | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Enc Time[%] | 98% | | | 98% | | |
| Dec Time[%] | 96% | | | 94% | | |

Table 9. Modified clipping in HM4.0 (internal bit-depth is 12 bits)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  | | --- | --- | --- | --- | |  | **All Intra HE** | | | |  | Y | U | V | | Class A | 0.0% | 0.1% | 0.1% | | Class B | 0.0% | 0.0% | 0.0% | | Class C | 0.0% | 0.0% | 0.0% | | Class D | 0.0% | 0.0% | 0.0% | | Class E | 0.0% | 0.0% | 0.0% | | Class F |  |  |  | | **Overall** | 0.0% | 0.0% | 0.0% | |  | 0.0% | 0.0% | 0.0% | | Enc Time[%] | #NUM! | | | | Dec Time[%] | 98% | | | |  |  |  |  | |  | **Random Access HE** | | | |  | Y | U | V | | Class A | 0.0% | 0.9% | 0.1% | | Class B | 0.0% | -0.1% | 0.1% | | Class C | 0.0% | 0.0% | 0.0% | | Class D | 0.0% | 0.1% | 0.0% | | Class E |  |  |  | | Class F |  |  |  | | **Overall** | 0.0% | 0.2% | 0.1% | |  | 0.0% | 0.2% | 0.1% | | Enc Time[%] | 99% | | | | Dec Time[%] | 99% | | | | |  |  |  |  | | --- | --- | --- | --- | |  | **Low delay B HE** | | | |  | Y | U | V | | Class A |  |  |  | | Class B | 0.0% | 0.0% | 0.0% | | Class C | 0.0% | 0.1% | 0.1% | | Class D | 0.0% | -0.1% | 0.2% | | Class E | 0.1% | -0.2% | -0.1% | | Class F |  |  |  | | **Overall** | 0.0% | 0.0% | 0.1% | |  | 0.0% | -0.1% | 0.1% | | Enc Time[%] | 99% | | | | Dec Time[%] | 99% | | | |  |  |  |  | |  | **Low delay P HE** | | | |  | Y | U | V | | Class A |  |  |  | | Class B | 0.0% | 0.1% | 0.0% | | Class C | 0.0% | -0.2% | 0.1% | | Class D | -0.1% | 0.1% | 0.2% | | Class E | 0.0% | -0.2% | 0.7% | | Class F |  |  |  | | **Overall** | 0.0% | 0.0% | 0.2% | |  | 0.0% | 0.0% | 0.3% | | Enc Time[%] | 99% | | | | Dec Time[%] | 98% | | | |

Table 10. Modified clipping with F251 (internal bit-depth is 12 bits)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  | | --- | --- | --- | --- | |  | **All Intra HE** | | | |  | Y | U | V | | Class A | 0.0% | 0.2% | 0.0% | | Class B | 0.0% | 0.0% | 0.0% | | Class C | 0.0% | 0.0% | 0.0% | | Class D | 0.0% | 0.0% | 0.0% | | Class E | 0.0% | 0.0% | 0.0% | | Class F |  |  |  | | **Overall** | 0.0% | 0.0% | 0.0% | |  | 0.0% | 0.0% | 0.0% | | Enc Time[%] | 100% | | | | Dec Time[%] | 99% | | | |  |  |  |  | |  | **Random Access HE** | | | |  | Y | U | V | | Class A | 0.0% | 0.0% | 0.1% | | Class B | 0.0% | -0.1% | 0.0% | | Class C | 0.0% | 0.0% | 0.0% | | Class D | 0.0% | 0.0% | 0.0% | | Class E |  |  |  | | Class F |  |  |  | | **Overall** | 0.0% | 0.0% | 0.0% | |  | 0.0% | 0.0% | 0.0% | | Enc Time[%] | 99% | | | | Dec Time[%] | 99% | | | | |  |  |  |  | | --- | --- | --- | --- | |  | **Low delay B HE** | | | |  | Y | U | V | | Class A |  |  |  | | Class B | 0.0% | -0.1% | 0.2% | | Class C | 0.0% | 0.3% | 0.1% | | Class D | 0.0% | -0.1% | 0.3% | | Class E | -0.1% | -0.3% | 0.1% | | Class F |  |  |  | | **Overall** | 0.0% | 0.0% | 0.2% | |  | 0.0% | 0.0% | 0.2% | | Enc Time[%] | 99% | | | | Dec Time[%] | 97% | | | |  |  |  |  | |  | **Low delay P HE** | | | |  | Y | U | V | | Class A |  |  |  | | Class B | 0.0% | -0.1% | -0.3% | | Class C | 0.0% | 0.0% | 0.0% | | Class D | 0.0% | -0.1% | 0.0% | | Class E | -0.1% | -0.7% | 0.1% | | Class F |  |  |  | | **Overall** | 0.0% | -0.2% | -0.1% | |  | 0.0% | -0.2% | -0.1% | | Enc Time[%] | 98% | | | | Dec Time[%] | 96% | | | |

# Conclusions

We propose to the following changes in HM5.0 and WD:

* Specify the size of both 3 temporal buffers used in HEVC inverse transform as 16 bits.
* Specify precision of intermediate calculation as 32 bits.
* Specify that output of dequantizer shall be clipped to the range –Max1…Max1
* Specify that output of column inverse transform shall be clipped to the range –Max2…Max2
* Specify that no clipping in needed after second (row) inverse transform.
* Specify that values Max1 and Max2 as defined by formulas (9)-(10) in this contribution.

Proposed changes guarantee no overflow of both 32 bits register and 16 bits temporal buffers with no more than 2 clip operations per pixel. There is no performance change under common test conditions and under-noise level performance variation if internal bit depth is set to 12.

# References

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# Patent rights declaration(s)

**Samsung Electronics Co., Ltd. may have current or pending patent rights relating to the technology described in this contribution and, conditioned on reciprocity, is prepared to grant licenses under reasonable and non-discriminatory terms as necessary for implementation of the resulting ITU-T Recommendation | ISO/IEC International Standard (per box 2 of the ITU-T/ITU-R/ISO/IEC patent statement and licensing declaration form).**

**Appendix A:** Transform matrices for HM4.0 core transform.

DST: =

{

{29, 55, 74, 84},

{74, 74, 0 , -74},

{84, -29, -74, 55},

{55, -84, 74, -29},

};

=

{

{ 64, 64, 64, 64},

{ 83, 36,-36,-83},

{ 64,-64,-64, 64},

{ 36,-83, 83,-36}

};

=

{

{ 64, 64, 64, 64, 64, 64, 64, 64},

{ 89, 75, 50, 18,-18,-50,-75,-89},

{ 83, 36,-36,-83,-83,-36, 36, 83},

{ 75,-18,-89,-50, 50, 89, 18,-75},

{ 64,-64,-64, 64, 64,-64,-64, 64},

{ 50,-89, 18, 75,-75,-18, 89,-50},

{ 36,-83, 83,-36,-36, 83,-83, 36},

{ 18,-50, 75,-89, 89,-75, 50,-18}

};

=

{

{ 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64},

{ 90, 87, 80, 70, 57, 43, 25, 9, -9,-25,-43,-57,-70,-80,-87,-90},

{ 89, 75, 50, 18,-18,-50,-75,-89,-89,-75,-50,-18, 18, 50, 75, 89},

{ 87, 57, 9,-43,-80,-90,-70,-25, 25, 70, 90, 80, 43, -9,-57,-87},

{ 83, 36,-36,-83,-83,-36, 36, 83, 83, 36,-36,-83,-83,-36, 36, 83},

{ 80, 9,-70,-87,-25, 57, 90, 43,-43,-90,-57, 25, 87, 70, -9,-80},

{ 75,-18,-89,-50, 50, 89, 18,-75,-75, 18, 89, 50,-50,-89,-18, 75},

{ 70,-43,-87, 9, 90, 25,-80,-57, 57, 80,-25,-90, -9, 87, 43,-70},

{ 64,-64,-64, 64, 64,-64,-64, 64, 64,-64,-64, 64, 64,-64,-64, 64},

{ 57,-80,-25, 90, -9,-87, 43, 70,-70,-43, 87, 9,-90, 25, 80,-57},

{ 50,-89, 18, 75,-75,-18, 89,-50,-50, 89,-18,-75, 75, 18,-89, 50},

{ 43,-90, 57, 25,-87, 70, 9,-80, 80, -9,-70, 87,-25,-57, 90,-43},

{ 36,-83, 83,-36,-36, 83,-83, 36, 36,-83, 83,-36,-36, 83,-83, 36},

{ 25,-70, 90,-80, 43, 9,-57, 87,-87, 57, -9,-43, 80,-90, 70,-25},

{ 18,-50, 75,-89, 89,-75, 50,-18,-18, 50,-75, 89,-89, 75,-50, 18},

{ 9,-25, 43,-57, 70,-80, 87,-90, 90,-87, 80,-70, 57,-43, 25, -9}

};

=

{

{ 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64, 64},

{ 90, 90, 88, 85, 82, 78, 73, 67, 61, 54, 46, 38, 31, 22, 13, 4, -4,-13,-22,-31,-38,-46,-54,-61,-67,-73,-78,-82,-85,-88,-90,-90},

{ 90, 87, 80, 70, 57, 43, 25, 9, -9,-25,-43,-57,-70,-80,-87,-90,-90,-87,-80,-70,-57,-43,-25, -9, 9, 25, 43, 57, 70, 80, 87, 90},

{ 90, 82, 67, 46, 22, -4,-31,-54,-73,-85,-90,-88,-78,-61,-38,-13, 13, 38, 61, 78, 88, 90, 85, 73, 54, 31, 4,-22,-46,-67,-82,-90},

{ 89, 75, 50, 18,-18,-50,-75,-89,-89,-75,-50,-18, 18, 50, 75, 89, 89, 75, 50, 18,-18,-50,-75,-89,-89,-75,-50,-18, 18, 50, 75, 89},

{ 88, 67, 31,-13,-54,-82,-90,-78,-46, -4, 38, 73, 90, 85, 61, 22,-22,-61,-85,-90,-73,-38, 4, 46, 78, 90, 82, 54, 13,-31,-67,-88},

{ 87, 57, 9,-43,-80,-90,-70,-25, 25, 70, 90, 80, 43, -9,-57,-87,-87,-57, -9, 43, 80, 90, 70, 25,-25,-70,-90,-80,-43, 9, 57, 87},

{ 85, 46,-13,-67,-90,-73,-22, 38, 82, 88, 54, -4,-61,-90,-78,-31, 31, 78, 90, 61, 4,-54,-88,-82,-38, 22, 73, 90, 67, 13,-46,-85},

{ 83, 36,-36,-83,-83,-36, 36, 83, 83, 36,-36,-83,-83,-36, 36, 83, 83, 36,-36,-83,-83,-36, 36, 83, 83, 36,-36,-83,-83,-36, 36, 83},

{ 82, 22,-54,-90,-61, 13, 78, 85, 31,-46,-90,-67, 4, 73, 88, 38,-38,-88,-73, -4, 67, 90, 46,-31,-85,-78,-13, 61, 90, 54,-22,-82},

{ 80, 9,-70,-87,-25, 57, 90, 43,-43,-90,-57, 25, 87, 70, -9,-80,-80, -9, 70, 87, 25,-57,-90,-43, 43, 90, 57,-25,-87,-70, 9, 80},

{ 78, -4,-82,-73, 13, 85, 67,-22,-88,-61, 31, 90, 54,-38,-90,-46, 46, 90, 38,-54,-90,-31, 61, 88, 22,-67,-85,-13, 73, 82, 4,-78},

{ 75,-18,-89,-50, 50, 89, 18,-75,-75, 18, 89, 50,-50,-89,-18, 75, 75,-18,-89,-50, 50, 89, 18,-75,-75, 18, 89, 50,-50,-89,-18, 75},

{ 73,-31,-90,-22, 78, 67,-38,-90,-13, 82, 61,-46,-88, -4, 85, 54,-54,-85, 4, 88, 46,-61,-82, 13, 90, 38,-67,-78, 22, 90, 31,-73},

{ 70,-43,-87, 9, 90, 25,-80,-57, 57, 80,-25,-90, -9, 87, 43,-70,-70, 43, 87, -9,-90,-25, 80, 57,-57,-80, 25, 90, 9,-87,-43, 70},

{ 67,-54,-78, 38, 85,-22,-90, 4, 90, 13,-88,-31, 82, 46,-73,-61, 61, 73,-46,-82, 31, 88,-13,-90, -4, 90, 22,-85,-38, 78, 54,-67},

{ 64,-64,-64, 64, 64,-64,-64, 64, 64,-64,-64, 64, 64,-64,-64, 64, 64,-64,-64, 64, 64,-64,-64, 64, 64,-64,-64, 64, 64,-64,-64, 64},

{ 61,-73,-46, 82, 31,-88,-13, 90, -4,-90, 22, 85,-38,-78, 54, 67,-67,-54, 78, 38,-85,-22, 90, 4,-90, 13, 88,-31,-82, 46, 73,-61},

{ 57,-80,-25, 90, -9,-87, 43, 70,-70,-43, 87, 9,-90, 25, 80,-57,-57, 80, 25,-90, 9, 87,-43,-70, 70, 43,-87, -9, 90,-25,-80, 57},

{ 54,-85, -4, 88,-46,-61, 82, 13,-90, 38, 67,-78,-22, 90,-31,-73, 73, 31,-90, 22, 78,-67,-38, 90,-13,-82, 61, 46,-88, 4, 85,-54},

{ 50,-89, 18, 75,-75,-18, 89,-50,-50, 89,-18,-75, 75, 18,-89, 50, 50,-89, 18, 75,-75,-18, 89,-50,-50, 89,-18,-75, 75, 18,-89, 50},

{ 46,-90, 38, 54,-90, 31, 61,-88, 22, 67,-85, 13, 73,-82, 4, 78,-78, -4, 82,-73,-13, 85,-67,-22, 88,-61,-31, 90,-54,-38, 90,-46},

{ 43,-90, 57, 25,-87, 70, 9,-80, 80, -9,-70, 87,-25,-57, 90,-43,-43, 90,-57,-25, 87,-70, -9, 80,-80, 9, 70,-87, 25, 57,-90, 43},

{ 38,-88, 73, -4,-67, 90,-46,-31, 85,-78, 13, 61,-90, 54, 22,-82, 82,-22,-54, 90,-61,-13, 78,-85, 31, 46,-90, 67, 4,-73, 88,-38},

{ 36,-83, 83,-36,-36, 83,-83, 36, 36,-83, 83,-36,-36, 83,-83, 36, 36,-83, 83,-36,-36, 83,-83, 36, 36,-83, 83,-36,-36, 83,-83, 36},

{ 31,-78, 90,-61, 4, 54,-88, 82,-38,-22, 73,-90, 67,-13,-46, 85,-85, 46, 13,-67, 90,-73, 22, 38,-82, 88,-54, -4, 61,-90, 78,-31},

{ 25,-70, 90,-80, 43, 9,-57, 87,-87, 57, -9,-43, 80,-90, 70,-25,-25, 70,-90, 80,-43, -9, 57,-87, 87,-57, 9, 43,-80, 90,-70, 25},

{ 22,-61, 85,-90, 73,-38, -4, 46,-78, 90,-82, 54,-13,-31, 67,-88, 88,-67, 31, 13,-54, 82,-90, 78,-46, 4, 38,-73, 90,-85, 61,-22},

{ 18,-50, 75,-89, 89,-75, 50,-18,-18, 50,-75, 89,-89, 75,-50, 18, 18,-50, 75,-89, 89,-75, 50,-18,-18, 50,-75, 89,-89, 75,-50, 18},

{ 13,-38, 61,-78, 88,-90, 85,-73, 54,-31, 4, 22,-46, 67,-82, 90,-90, 82,-67, 46,-22, -4, 31,-54, 73,-85, 90,-88, 78,-61, 38,-13},

{ 9,-25, 43,-57, 70,-80, 87,-90, 90,-87, 80,-70, 57,-43, 25, -9, -9, 25,-43, 57,-70, 80,-87, 90,-90, 87,-80, 70,-57, 43,-25, 9},

{ 4,-13, 22,-31, 38,-46, 54,-61, 67,-73, 78,-82, 85,-88, 90,-90, 90,-90, 88,-85, 82,-78, 73,-67, 61,-54, 46,-38, 31,-22, 13, -4}

};

**Appendix B:** Transform matrices for F251 core transform.

DST: =

{

{1856, 3520, 4736, 5376},

{4736, 4736, 0, -4736},

{5376, -1856, -4736, 3520},

{3520, -5376, 4736, -1856},};

=

{

{ 4096, 4096, 4096, 4096},

{ 5312, 2304,-2304,-5312 },

{ 4096,-4096,-4096, 4096},

{ 2304,-5312, 5312,-2304 }

};

=

{

{ 4096, 4096, 4096, 4096, 4096, 4096, 4096, 4096,},

{ 5696, 4800, 3264, 1088,-1088,-3264,-4800,-5696,},

{ 5312, 2304,-2304,-5312,-5312,-2304, 2304, 5312,},

{ 4770,-1080,-5670,-3240, 3240, 5670, 1080,-4770,},

{ 4096,-4096,-4096, 4096, 4096,-4096,-4096, 4096,},

{ 3240,-5670, 1080, 4770,-4770,-1080, 5670,-3240,},

{ 2304,-5312, 5312,-2304,-2304, 5312,-5312, 2304,},

{ 1088,-3264, 4800,-5696, 5696,-4800, 3264,-1088,},};

=

{

{ 4096, 4096, 4096, 4096, 4096, 4096, 4096, 4096, 4096, 4096, 4096, 4096, 4096, 4096, 4096, 4096, },

{ 5760, 5504, 5120, 4480, 3712, 2688, 1664, 512, -512, -1664, -2688, -3712, -4480, -5120, -5504, -5760, },

{ 5696, 4800, 3264, 1088, -1088, -3264, -4800, -5696, -5696, -4800, -3264, -1088, 1088, 3264, 4800, 5696, },

{ 5543, 3568, 619, -2802, -5096, -5747, -4506, -1639, 1639, 4506, 5747, 5096, 2802, -619, -3568, -5543, },

{ 5312, 2304, -2304, -5312, -5312, -2304, 2304, 5312, 5312, 2304, -2304, -5312, -5312, -2304, 2304, 5312, },

{ 5167, 474, -4379, -5528, -1806, 3773, 5728, 2591, -2591, -5728, -3773, 1806, 5528, 4379, -474, -5167, },

{ 4770, -1080, -5670, -3240, 3240, 5670, 1080, -4770, -4770, 1080, 5670, 3240, -3240, -5670, -1080, 4770, },

{ 4410, -2700, -5490, 540, 5760, 1710, -5040, -3690, 3690, 5040, -1710, -5760, -540, 5490, 2700, -4410, },

{ 4096, -4096, -4096, 4096, 4096, -4096, -4096, 4096, 4096, -4096, -4096, 4096, 4096, -4096, -4096, 4096, },

{ 3690, -5040, -1710, 5760, -540, -5490, 2700, 4410, -4410, -2700, 5490, 540, -5760, 1710, 5040, -3690, },

{ 3240, -5670, 1080, 4770, -4770, -1080, 5670, -3240, -3240, 5670, -1080, -4770, 4770, 1080, -5670, 3240, },

{ 2591, -5728, 3773, 1806, -5528, 4379, 474, -5167, 5167, -474, -4379, 5528, -1806, -3773, 5728, -2591, },

{ 2304, -5312, 5312, -2304, -2304, 5312, -5312, 2304, 2304, -5312, 5312, -2304, -2304, 5312, -5312, 2304, },

{ 1639, -4506, 5747, -5096, 2802, 619, -3568, 5543, -5543, 3568, -619, -2802, 5096, -5747, 4506, -1639, },

{ 1088, -3264, 4800, -5696, 5696, -4800, 3264, -1088, -1088, 3264, -4800, 5696, -5696, 4800, -3264, 1088, },

{ 512, -1664, 2688, -3712, 4480, -5120, 5504, -5760, 5760, -5504, 5120, -4480, 3712, -2688, 1664, -512, },

};

=

{

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