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| **Joint Collaborative Team on Video Coding (JCT-VC)**  **of ITU-T SG16 WP3 and ISO/IEC JTC1/SC29/WG11**  5th Meeting: Geneva, CH, 16-23 March, 2011 | Document: JCTVC-E341 |

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| *Title:* | **Approximation Quality of Integer Transforms** | | |
| *Status:* | Input Document to JCT-VC | | |
| *Purpose:* | Proposal | | |
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# Abstract

For the core transform of HEVC, integer approximations of the DCT are considered. Some of the proposed transforms relate to the butterfly structure proposed by Chen in 1977. Some others rely on matrix definitions. For orthogonal transform approximations, a MSE-based measure has been proposed by de Queiroz to evaluate the approximation quality of the approximation under discussion. The measure is proposed as a potential aid in the transform selection process and the approximation quality of some integer transform approximations is discussed. It is noted that non-negligible approximation errors may be an indicator for impact on the visual reconstruction quality of the employed transform.

# Introduction

The DCT-II of size has been a well-established and integral part of the video coding specifications in both ITU-T and MPEG. It is theoretically founded by the derivation via the KLT of a AR(1) processes with ; the regular structure of its base vectors is exploited in fast implementations. Its properties and impact on the visual quality of the reconstructed video has been broadly studied.

For the core transform of HEVC –like H.264/AVC– integer approximations of the DCT are considered. In the development process of H.264/AVC, integer transforms of size to have been proposed in the context of Adaptive Block-size Transforms (ABT), see e.g. [2],[3],[4].

Some of the proposed transforms for HEVC relate to the butterfly structure proposed by Chen in 1977. Some others rely on matrix definitions. The suitability of the transform proposals is mainly evaluated based on the rate-distortion performance in the HM-software and the implementation (and description) complexity of the transform and the corresponding quantization processes. Since the specified transform has a non-negligible impact on the visual reconstruction quality, careful visual assessment of the transform under consideration is advisable.

In this document it is proposed to apply an objective MSE-based evaluation criterion for the approximation quality of the transforms under consideration. While indicating the proximity of a DCT approximation to the DCT itself, the criterion may also provide some confidence on the quality of the reconstructed quantized signal. Proximity here might indicate the reconstructed transform quantization error to be similar to the corresponding error by a ‘true’ DCT.

# Approximation Error

For the design of unitary transform approximations, a MSE-based measure has been proposed by de Queiroz in 1998 to evaluate the approximation quality of the approximation under discussion [1]. The measure is based on the variance of the transform output error for a given signal statistics, and thereby appears to be well suited for the given context.

Let , with a signal oflength *N*, the DCT matrix, and the approximation of the DCT matrix. Note that is a normalized orthogonal transform matrix, with appropriate normalization of the transform base vectors according to their norms. The proposed criterion is given as

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The approximation error depends on the difference of the approximation and the DCT matrix in connection with the auto correlation of the input signal. As with the coding gain, an AR(1) model with may be assumed for evaluation of the transform approximation quality.

It should be noted that non-negligible approximation error may indicate issues that could have strong impact on the visual reconstruction quality of the employed transform. Such visual impact may potentially be hard to control and difficult to mitigate in a subsequent (loop-)filtering process.

# Evaluation of Integer Matrix Approximations of the DCT

## DCT Symmetry

Due to the well-known symmetry features of the DCT matrix, coefficients are required to construct a DCT matrix. The even base vectors of a DCT matrix can be derived by even mirroring of the *N*-size base vectors, the odd based vectors exhibit odd symmetry. These features can be exploited for the construction of integer approximations of the DCT matrix.

Let the structure of the matrix be

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Let the base vector denote the even rows of the matrix, and let denote the odd rows. Accordingly, the matrix reuses these base vectors with the coefficients on the even positions and 4 additional odd base vectors with coefficients on the odd positions. For the and the matrices, base vectors , with coefficients and are introduced, respectively.

Due to the symmetry of the transform matrix, the base vectors , …, are mutually orthogonal. Within each class , the scalar product of the base vectors defines the conditions for suitable coefficients.

Based on the stated features, integer coefficients can be derived which fulfill the given constraints. In combination with the coding gain, the approximation error may serve as a performance measure for the selection of suitable transform matrix coefficients. Additional constraints may be given by implementation requirements and complexity considerations.

## Integer Transform Examples

In the following tables examples for integer DCT matrix approximations are given. The resulting coding gain and the approximation error are indicated for evaluation. An AR(1) process with is assumed as the signal model.

**transform: (DCT: )**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | [dB] |  |
| 13 | 17 | 7 | 676 | 676 |  |  |
| 1 | 2 | 1 | 4 | 10 |  |  |

**transform: (DCT: )**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  | [dB] |  |
| 17 | 23 | 7 | 24 | 20 | 12 | 6 | 2312 | 2312 | 2312 |  |  |
| 1 | 29 | 12 | 25 | 21 | 14 | 5 | 8 | 3940 | 2574 |  |  |
| 1 | 2 | 1 | 12 | 10 | 6 | 3 | 8 | 20 | 578 |  |  |

**transform: (DCT: )**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | [dB] |  |
| 17 | 23 | 7 | 24 | 20 | 12 | 6 | 81 | 80 | 74 | 67 | 53 | 40 | 23 | 13 |  |  |
| 1 | 29 | 12 | 25 | 21 | 14 | 5 | 81 | 80 | 74 | 67 | 53 | 40 | 23 | 13 |  |  |

# Applicability to Non-Orthogonal Approximations

While matrices obeying the aforementioned constraints provide inherently orthogonal matrices, approximations based on the Chen butterfly structure are not necessarily orthogonal. These transforms may be considered close to but not exactly orthogonal.

Since the proposed criterion relies on the orthogonality of the considered matrices, it cannot be applied for non-orthogonal approximations.

If the orthogonality condition is satisfied, the approximations can be transferred into a matrix representation and the proposed criterion can be used.

# Conclusions

An evaluation criterion for orthogonal integer approximations of the DCT matrix based on the MSE has been presented. This approximation error measure has been proposed in literature by de Queiroz. In combination with the well-known transform coding gain it can be considered as a suitable tool for the assessment and comparison of different integer DCT approximations.

The approximation error gives an indication of the proximity of the transformed signal to the one transformed by the DCT. It thereby provides some confirmation that the deviation from the visual reconstruction quality of the DCT may be limited. Careful visual assessment of the reconstruction quality appears advisable nonetheless.

It should be noted that a weak approximation quality may indicate issues with the reconstruction quality. These issues may be difficult to mitigate in subsequent filtering processes.

# References

1. R.L. de Queiroz, “On Unitary Transform Approximations,” IEEE Signal Processing Letters, vol. 5, no. 2, pp. 46–47, Feb. 1998.
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3. M. Wien and S. Sun, “ICT comparison for adaptive block transforms,” Doc. VCEG-L12, ITU-T SG16/Q6 VCEG, Eibsee, Germany, 12th meeting, Jan. 2001.
4. M. Wien and A. Dahlhoff, “16 bit adaptive block size transforms,” Doc. JVT-C107, Joint Video Team (JVT) of ISO/IEC MPEG & ITU-T VCEG, Fairfax, VA, USA, 3rd Meeting, May 2002.

# Patent rights declaration(s)

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